

Vertical Soil Arching and TerraFlex

Reference: Soil Engineering, 4th Edition, Spangler and Handy, 1982, Harper & Row.

Introduction: Pipes, conduits, small tunnels, and other buried structures known as underground or buried conduits experience stresses from the overlying soil as well as surcharges or other loads on the surface. These stresses all are affected by a property known as vertical soil arching or simply, “soil arching.”

The basic concept of soil arching is that a part of the weight of the soil (and any surcharge) is transferred between the soil “prism” over the conduit and adjacent soil “prisms”. This phenomenon can lead to stresses that are greater than the total weight of the overlying soil plus the surface loads (negative arching), or it can result in loads that are significantly less (positive Arching or “arching”). Therefore, it is very important to take the property of arching into account when designing an underground conduit.

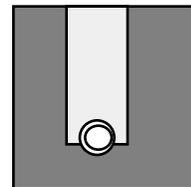
This paper will describe the theory of soil arching, and provide analysis that shows the benefit of inducing soil arching with a synthetic compressible inclusion, specifically TerraFlex.

The Theory of Soil Arching:

There are two basic types of buried conduits, classified according to their placement relative to the original ground surface.

A **Ditch conduit**, shown at right, illustrates this construction technique. This is by far the most benign approach, as the forces on the buried structure will be the least. It is fairly common for small, shallow conduits, such as for minor utility lines.

Natural Ground Surface



Ditch
Conduit

The load on the conduit, in force per foot, can be found by the relationship

$$W = C_d \cdot \gamma \cdot B_d^2$$

Equation 1: Load per unit length of Conduit, with Ditch Coefficient

Where

W is the load (force) per unit length of the conduit,

C_d is the “Ditch Coefficient”

γ is the total unit weight of the fill over the conduit, and

B_d is the width of the ditch.

Without any arching effect, the factor C_d would be simply H/B_d , and the equation above would reduce to

$$W = \gamma \cdot H \cdot B_d$$

Equation 2: Load per unit length of conduit with no arching effects

The effect of arching can be modeled by a simple linear differential equation which provides a ditch coefficient that accounts for arching as,

$$C_d = \frac{1 - e^{-2 \cdot K_a \cdot \mu' \cdot \left(\frac{H}{B_d}\right)}}{2 \cdot K_a \cdot \mu'}$$

Equation 3: Ditch Coefficient for “Complete Ditch” Condition

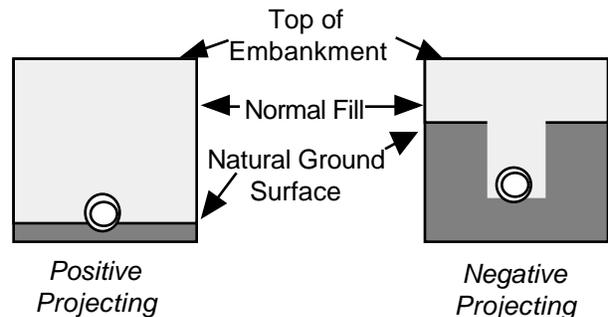
Where

K_a = the ratio of the active lateral unit pressure to the vertical unit pressure (commonly referred to as the “coefficient of active earth pressure”) and $\mu' = \tan \phi$ where ϕ is the friction angle between the ditch fill and the natural soil

What this equation tells us is that the greater the height of the ditch compared to it’s width, the lower the stress on the buried conduit.

The factors “ K_a ” and “ μ ” are lumped together in this analysis, since they both derive from the same fundamental property of the soil (the angle of internal friction).

Conduits which are placed beneath areas where the grade is raised as part of construction are known as “**projecting conduits**.” This a more general case in practice, representing the situation with road and rail embankments passing over utility lines. There are two variations: the Positive Projecting and the Negative Projecting conduit, shown to the right.



The **positive projecting conduit** is the most common type of projecting conduit, and is the simplest to construct, but may actually result in loads that are GREATER than the weight of the overlying soil. To determine what the stress is, accounting for arching effects (which can be negative arching), it is necessary to study a property known as the “settlement ratio.”

The settlement ratio for a positive projecting conduit is given as:

$$r_{sd} = \frac{(s_m + s_g) - (s_f + d_c)}{s_m}$$

Equation 4: Settlement Ratio for Positively Projecting conduit

Where:

r_{sd} = settlement ratio (dimensionless)

s_m = compression of the soil of height $p \times B_c$

s_g = settlement of the natural ground surface adjacent to the conduit under weight of fill

s_f = settlement of the conduit into its foundation

d_c = shortening of the vertical height of the conduit.

In the definition of s_m , there is a term “p” which is the “projection ratio.” It is simply the distance from the natural ground surface to what is known as the “critical plane” located at the very top of the conduit, expressed as a factor of the conduit width, B_c .

The “critical plane” is that horizontal plane above the conduit, the movement of which relative to the top of the conduit determines what is going on between the soil column or (or “prism”) above the conduit and the soil columns (Prisms) on either side of it- If this plane moves further down than the top of the conduit (resulting in a POSITIVE settlement ratio), the conduit will experience greater loads than expected, due to the effects of negative arching. If this plane moves LESS than the top of the conduit (resulting in a NEGATIVE settlement ratio), then the loads on the conduit will be less than expected, due to positive arching.

The shear effects in the soil causing this arching behavior extend above the conduit to a height known as the “Height of equal settlement” or H_e . If this height is above the actual embankment height, there will be a dip or a hump on the surface from the uneven settlements above the conduit caused by the negative or positive arching. If H_e is computed to be below the surface, then the surface will be unaffected by the arching effects taking place beneath the surface.

In order to compute H_e and the ditch coefficient (C_d) for positive projecting conduits, a formula is derived by equating an expression for the sum of the total strain in the prism above the conduit plus the settlement of the top of the conduit to a similar expression for the sum of the total strain in the exterior prism plus the settlement of the critical plane. This formula, which is best solved numerically, is:

$$\frac{\left[\frac{1}{2 \cdot k \cdot \mu} - \left(\frac{H}{B} - \frac{H_e}{B_c} \right) - \frac{r_{sd} P}{3} \right] \cdot e^{-2 \cdot k \cdot \mu \cdot \left(\frac{H_e}{B_c} \right)} - 1}{-2 \cdot k \cdot \mu} = \frac{1}{2} \cdot \left(\frac{H_e}{B_c} \right)^2 - \frac{r_{sd} P}{3} \cdot \left(\frac{H - H_e}{B_c} \right) \cdot e^{-2 \cdot k \cdot \mu \cdot \left(\frac{H_e}{B_c} \right)} - \frac{1}{2 \cdot k \cdot \mu} \cdot \frac{H_e}{B_c} + \frac{H \cdot H_e}{B_c^2} = r_{sd} P \cdot \frac{H}{B_c}$$

Equation 5: For numerical solution of H_e , given settlement ratio

Having estimated H_e , there is a derivation that shows the ditch coefficient to be:

$$C_d = \frac{1 - e^{-2 \cdot K_a \cdot \mu' \cdot \left(\frac{H_e}{B_d} \right)}}{2 \cdot K_a \cdot \mu'} + \left(\frac{H}{B_c} - \frac{H_e}{B_c} \right) \cdot e^{-2 \cdot K_a \cdot \mu' \cdot \left(\frac{H_e}{B_c} \right)}$$

Equation 6: Computing Ditch Coefficient, given H_e

Following the format of Spangler and Handy, a numerical solution of these two equations gives a family of curves corresponding to a given projection ratio and settlement ratio, similar to what is shown on page 7.

The value for C_d calculated in this manner is always greater than the C_d calculated for the ditch conduit (equation 3). When the settlement ratio is exactly zero, there actually is no arching at all, and C_d becomes simply the linear relationship first described (see Equation 2):

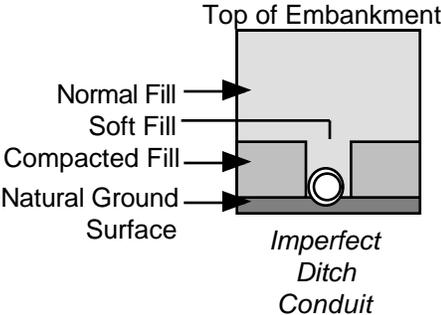
$$C_d = \frac{H}{B_d}$$

For the **negative projecting conduit**, where the conduit lies in a ditch excavated from the original ground level and then covered with backfill and fill to build an embankment, the procedure is the same, except the projection ratio is the depth of the ditch measured from the top of the conduit, expressed as a factor of the ditch width.

Analysis (and experience) indicates that positive projecting conduits tend to develop positive settlement ratios, which implies negative arching and thus amplifies the forces experienced by the conduit, whereas negative projecting conduits develop negative settlement ratios, where the forces on the conduit are attenuated by positive soil arching action.

At this point, it can be generally stated Ditch conduits and Negative Projecting conduits develop the least stress, and Positive Projecting conduits develop the most. For the case of projecting conduits, the most critical factor becomes the **settlement ratio**, which is a function of the conduit type and the how the fills are chosen and prepared, used over the conduit.

In order to ensure the settlement ratio is negative and the loads on the buried conduit are lower than the positively projecting case, a variant of the positively projecting conduit was developed to essentially get the benefit of the negatively projecting conduit in a situation that would normally result in a positively projecting conduit. This design is known as the “imperfect ditch conduit” and is shown at right. This is a positive projecting design, except that the soil prisms on either side of the conduit are compacted more than the soil prism above the conduit, making in effect a negatively projecting conduit.



The settlement ratio in this special case is given by

$$r_{sd} = \frac{s_g - (s_d + s_f + d_c)}{s_d}$$

Equation 7, settlement ratio for “imperfect Ditch”

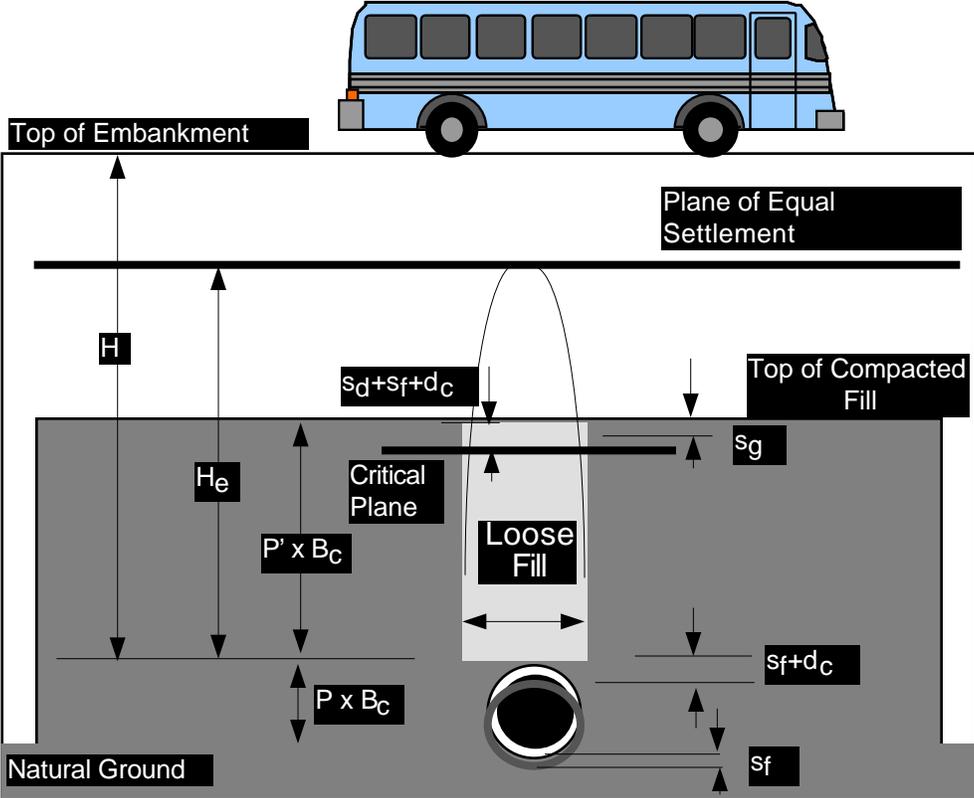
where

- s_g = settlement of surface of compacted soil
- s_d = compaction of fill in ditch, p/Bc in height
- s_f = settlement of conduit flow line
- d_c = shortening of conduit in feet

And the projection ratio is now the ratio of the ditch width (B_c) to it’s depth. This design and all it’s associated parameters is illustrated on the following page.

Although the advantages of the “imperfect ditch” type of conduit are clear, constructing one does present some practical difficulties. Chiefly among these is the establishment of the steep sided trench above the conduit, up to the desired height (projection ratio). The associated OSHA regulations for this type of construction can make the cost prohibitive. One solution has been the “modified imperfect ditch” technique where the soil over the the conduit is compacted all the way across the conduit up to the necessary height, and the trench is then excavated and filled with soft fill.

A further refinement to this “imperfect ditch” technique is to utilize a synthetic compressible inclusion in place of the loose fill. The compaction can take place adjacent to the inclusion, eliminating the need to install shoring. If the modified procedure is utilized where the ditch is excavated out of the compacted fill, the use of a lightweight compressible inclusion nearly eliminates the risk of cave in during the fill procedure, since no heavy machinery is required.



With this figure, recall that

$$C_d = \frac{1 - e^{-2 \cdot K_a \cdot \mu' \cdot \left(\frac{H_e}{B_d}\right)}}{2 \cdot K_a \cdot \mu'} + \left(\frac{H}{B_c} - \frac{H_e}{B_c}\right) \cdot e^{-2 \cdot K_a \cdot \mu' \cdot \left(\frac{H_e}{B_c}\right)}$$

Equation 6

And the term H_e , the plane of equal settlement will be determined by a numerical solution of **Equation 5** which is dependent upon the projection ratio “P” (which is proportional to the depth of the ditch), and the settlement ratio, r_{sd} given by,

$$r_{sd} = \frac{s_g - (s_d + s_f + d_c)}{s_d}$$

Equation 7

where

- s_g = settlement of surface of compacted soil
- s_d = compaction of fill in ditch, $p'B_c$ in height
- s_f = settlement of conduit flow line
- d_c = shortening of conduit in feet

Practical Example:

To illustrate the design method utilizing an imperfect ditch and inducing soil arching to minimize the load on the conduit, we look at the following problem:

Job Specifics:

Conduit Width = 5 feet.

Embankment Height = 40 feet

Soil Characteristics: $k\mu = 0.13$ (typical value for negatively projecting conduits)

Objectives:

Minimize Load on Conduit

Minimize Cost of Installation

Do not permit any deformations at the surface over the conduit.

Approach:

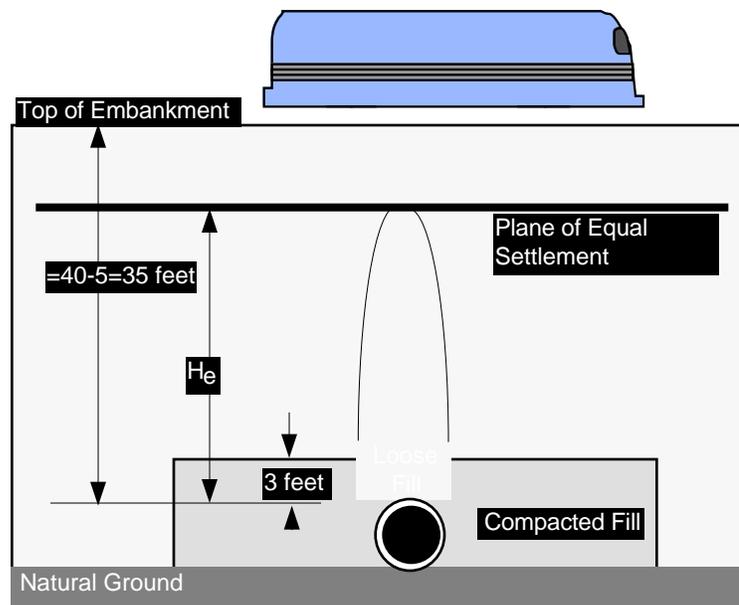
Minimize load on conduit calls for a design taking advantage of soil arching. We will design for a specific settlement ratio using compressible fills.

Minimizing cost of construction drives us to minimize the amount of soil compaction necessary, so the conduit will be designed to have a projection ratio of 1/2 (the sides of the artificial ditch will be 1/2 as tall as the diameter of the buried conduit, about three feet in this case).

In creating an imperfect ditch conduit, the conduit is placed on the natural ground surface, and then is buried in soil that is compacted up to the height required by the design, out to about three conduit widths to the side. In this case, the required volume of compacted soil required will be:

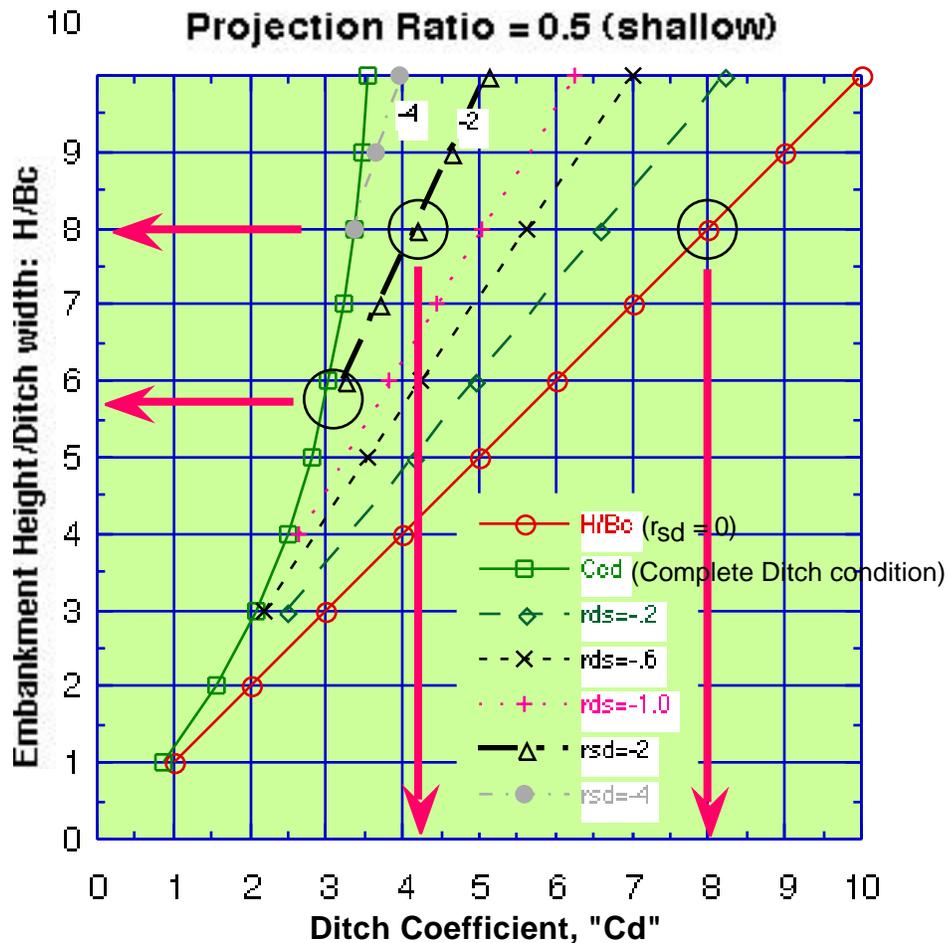
$5 + 3 = 8$ feet high, and
 $5 \times 3 \times 2 = 30$ feet wide.

Once the conduit has been buried in the necessary thickness of compacted soil, a trench (ditch) is excavated directly over the conduit, with the width of the conduit. This trench is then filled with a compressible fill, and then the rest of the embankment is built up to the necessary height utilizing normal fill methods. The embankment is illustrated to the right:



To avoid deformations at the surface of the conduit means that the design must result in a "plane of equal settlement" below the surface of the embankment.

When the “Ditch Coefficient” for a projection ratio of 0.5 is studied, the relationship between the settlement ratio, the ratio H/B_C and the ditch coefficient, C_D is given by the graph below.



The ratio H/B_C will be often constrained by the embankment design. The height of the embankment provides “H”, and the width of the conduit provides “ B_C .” In this specific problem case, this ratio equals $40 \div 5 = 8$.

What the designer can vary, however, is the settlement ratio. The graph reveals that if there was no soil arching (settlement ratio = 0), the Ditch Coefficient would be identical to the Height over the Trench Width, H/B_C . If the soil had an average unit weight of 124 pounds per cubic foot, this would correspond to a load of

$125 \text{ pcf} \times 35 \text{ feet of overlying soil} \times 5 \text{ foot diameter} = \mathbf{21,875 \text{ pounds load per foot.}}$

In this case of the “imperfect ditch”, the designer plans to have a significant negative “settlement ratio” which will induce soil arching to reduce the load on the conduit. In this case, if he designs to have a settlement ratio of -2, the load on the conduit is reduced by nearly half:

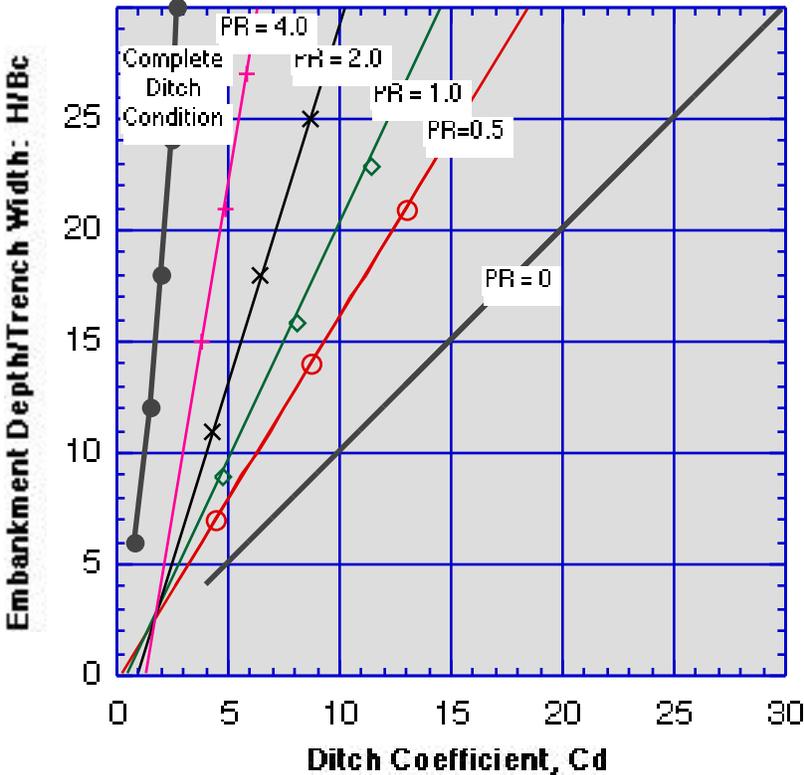
$125 \text{ pcf} \times 4.1 C_D \text{ (ditch coefficient)} \times (5 \text{ foot diameter})^2 = \mathbf{12,810 \text{ pounds per foot.}}$

As previously mentioned, the plane of equal settlement must lie below the surface of the embankment to avoid differential settlement over the ditch. The plane of equal settlement is determined by the intersection of the settlement ratio line with the "Complete Ditch" coefficient line. In this case, the plane of equal settlement occurs at an H/Bc = 6, or 30 feet, which is comfortably below the top of the embankment. There will be no settlement at the surface due to the effects of soil arching.

With a greater H/Bc, the soil arching effect is even greater. If the embankment was to be 50 feet high (H/Bc=10) instead of 40 feet high (H/Bc = 8), the designer could actually get even lower ditch coefficient by designing for a settlement ratio of -4.

Alternatively, the designer can assume a settlement ratio of -1, and increase the depth of the ditch (compacting a greater thickness of soil over the ditch). This can have a profound effect on the amount of load reduction experienced by the conduits, especially for very deeply buried conduits, as illustrated in the following chart:

31 Rsd=-1, Varying Projection Ratio



If the designer was working with a five foot wide conduit, as before, but was going to bury it under eighty feet of soil, this would indicate a H/Bc ratio of 16. Compacting the soil for a thickness of ten feet over the conduit gives a projection ratio of 2 (versus .5, as before). Now the relative loads will be:

No Arching = 125 pcf soil x 75 foot soil depth x 5 foot width = **46,875 pounds/ft**
 Arching = 125 x 6 Ditch Coefficient (Cd) x (5 foot)² = **18,750 pounds/ft**
LESS THAN HALF!

Designing for a Settlement Ratio/Choosing a Compressible Inclusion

In an imperfect ditch design, the settlement ratio has been given as:

$$r_{sd} = \frac{s_g - (s_d + s_f + d_c)}{s_d}$$

In this equation, the presence of a compressible inclusion effectively serves to increase the term d_c , which normally represents deformation of the conduit. It is important in setting up soil arching effects that the *bottom* of the column of soil in the trench deforms *more* than the *top*. Therefore, the compressible inclusion **MUST** be at the bottom of the “ditch,” up to a height that will give the needed settlement ratio.

Remember, if too much of the compressible inclusion is used, the pressure on the conduit may be lower, but the *plane of equal settlement* may end up being *above* the top of the embankment (imaginary, in other words), and there will settlement at the surface from the compression of the inclusion over the conduit.

Historically, the compressible inclusion used in this application has been baled hay, as documented in Spangler and Handy. Although projects utilizing hay have been generally successful, there are several problems with the use of hay or other organic materials (such as saw-dust) as a compressible inclusion:

1. Unpredictable properties. There is NO published data providing the compressibility of hay bales (or saw dust). If one were to develop this data, it would be highly variable, depending on the source and processes used to create the material. This makes it impossible for a designer to make accurate predictions about the settlement ratio and the resulting pressures on the conduit system, when utilizing this type of material.
2. Organic Decomposition. Organic compressible inclusions will decay over time which can result in the generation of explosive gasses, but more importantly, as these materials disappear, they leave a void above the conduit which can eventually fill due to raveling of the soil into the void which will, over time, reduce and even eliminate the effects of soil arching.

TerraFlex

TerraFlex is a synthetic compressible inclusion which effectively addresses all of the shortcomings previously mentioned related to organic compressible inclusions.

1. Testing. TerraFlex has been extensively tested for its stress strain properties under both rapid loading conditions as well as long term creep. This permits the engineer to accurately determine how much material will be necessary to attain the objectives of his design (the specific settlement ratio), which can result in savings in time and construction costs since the trench doesn't have to be over designed to account for the uncertainty associated with other materials.
2. It is inorganic. TerraFlex retains its properties indefinitely when buried, period. It does

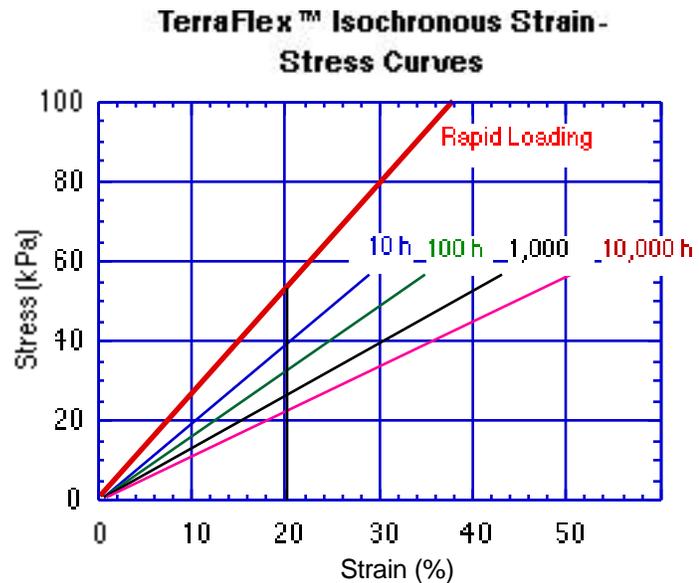
not decay or harden, nor does it release any contaminants to the surrounding soil.

3. It is an excellent insulator and can serve to protect buried conduits from freezing.

The following chart summarizes the salient properties of TerraFlex:

Property	Value
Density	0.77 lb/ft ³ (12kg/m ³)
R-Value	3.5 per inch (Dry)
Strength	See Chart to Right (Testing IAW ASTM 1621)

Protect TerraFlex Synthetic Compressible Inclusion from exposure to hydrocarbons, highly solvent extended mastics and coal tar pitch.



Summary

Virtually all conduits can be designed to benefit from the effects of soil arching. There is a well established methodology for doing this, which can be optimized to suit a range of design requirements. The most critical parameter in designing for soil arching is the “settlement ratio.”

The best material for use as a compressible inclusion to induce soil arching would be the synthetic compressible inclusion, **TerraFlex**.

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